Terminological Inconsistencies of "Elasticity of Substitution," "Substitution Curve," and "Marginal Rate of Substitution": A Correction

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Abstract

The present paper seeks to correct the inconsistencies of the three terms: (i) "elasticity of substitution" *(ES)*, (ii) "substitution curve" *(SC)*, and (iii) "marginal rate of substitution" *(MRS)*. Such inconsistencies persist since their introduction in the 1930s. The correction of such inconsistencies can be executed in terms of Konar's (2009, 2012) "isoquant substitution curve" *(IQSC)* and "isocost substitution curve" *(ICSC)*.

Keywords : **isoquant, isocost, substitution**

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Several authors have disclosed the ambiguity of the word "substitution" (e.g. Allen & Mishan, 1965; Folk & Wolfe, 1964). But no author has shown interest in identifying and correcting the inconsistencies of the three terms: (i) "elasticity of substitution" (*ES*), (ii) "substitution curve" (*SC*) and (iii) "marginal rate of substitution" (*MRS*). Such inconsistencies persist since their introduction in the 1930s. Hence, the objective of the present paper is to correct such inconsistencies.

Origin and Meaning of Substitution

The concept of "substitution" was introduced by the Greek scientist Archimedes (287-212 B.C.) of Syracuse in his *On the Sphere and Cylinder*. He wrote: "Further of unequal lines, unequal surfaces, and unequal solids, the greater exceeds the less by such a magnitude as, when added to itself, can be made to exceed any assigned magnitude among those, which are comparable with (it and with) one another" (Heath, Undated, p.4). The signification of substitution can be understood from the "law of substitution of similars" and the "axiom of substitution". The "law of substitution of similars" was applied in his economic theory as the "law of indifference" by Wicksteed (Robertson, 1951, p. 243). But the "axiom of substitution" can be defined from the economist's intuitive point of view - "Given a particular quantity of some commodity of which a person is desirous, it is always possible to find some quantity of other commodities sufficiently great to compensate him for loss of part of his consumption of the given commodity" (Chipman, 1960, p. 194). Stigler (1966, pp.25-26) emphasized that "there is no simple 'technical' measure of substitution: not only is it difficult to compare heterogeneous things (is radio a better substitute for television than for a theatre or a newspaper?), but substitutability varies with circumstances (a tractor is a substitute for a horse to a farmer, less so to a riding academy)". The process of substitution can be likened to the :

(i) Operation of a scissor ;

(ii) Process of exchange and ;

(iii) Process of chemical reaction.

In the first case, both the blades of a scissor function simultaneously in any operation. In the second case, both the sale and purchase of any commodity are subject to synchronization. And in the third case, both the oxidation and reduction occur simultaneously during the process of a chemical reaction. Thus, substitution is a process/phenomenon, in which both the inclusion/addition of something, and the exclusion/subtraction of another thing occur simultaneously.

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Substitution between any two variables L and K is of two types:

(i) Substitution of L for K, which can be symbolized by S_{LK} and ;

(ii) Substitution of K for L, which can be denoted by S_{KL}.

In the case of S_{LK}, L is injected, included, or added, while K is ejected, excluded, or ousted simultaneously. But in the case of S_{KL}, K is injected, included, or added, while L is ejected, excluded, or ousted simultaneously.

The concept of "substitution" is associated with the concept of "iso-z- function" (*IZF*) denoted by $Z = Z(L, K)$, where Z is a parameter and L and K are two variables. Along a given *IZF*, S_{LK} or S_{KL} is possible, if and only if the algebraic slope of the *IZF* is negative (i.e. $dK/dL<0$) irrespective of the curvature of the *IZF* (i.e. $d^2K/dL^2 \gtreqless 0$), provided that $d^2K/dL^2 \neq \infty$. Along a given *IZF*, which is negatively sloping, S_{LK} implies that an increase in L must be accompanied by a decrease in K in order to maintain the "isoness" of the *IZF* : $Z=Z(L, K)$. Further, S_{KL} refers to the synchronization of an increase in K and a decrease in L for keeping intact the parametric value of Z along a given *IZF*.

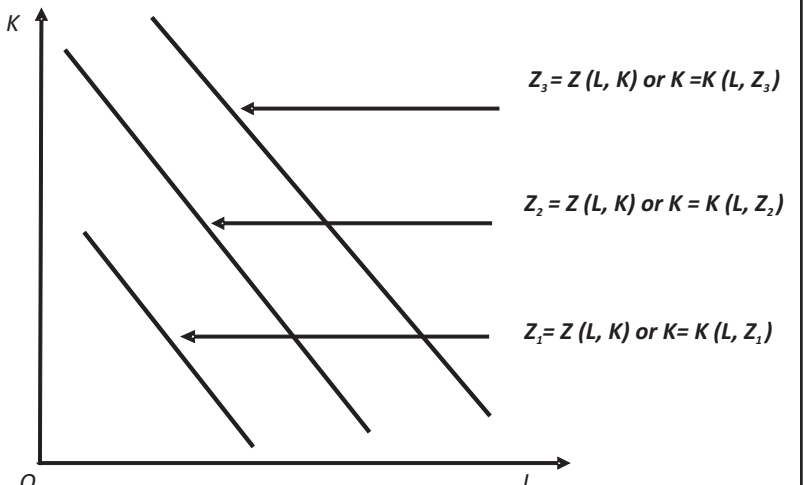
Origin and Meaning of Iso-Z-Function

The concept of "iso-z-funtion" *(IZF)* was introduced into economics in the name of "isoperimetric problem" by the Italian mathematician Paolo Frisi, the editor of the book *Meditazioni Sulla Economia Polotica* (1772), written by Pietro Verri (Robertson, 1949).

Though *IZF* is a multivariate function, yet the present analysis will be confined only to bi-variate *IZF*, which can be represented by $Z=Z(L,K)$, where Z is a parameter, but L and K are two variables. The nomenclature of *IZF* is determined by the name of the parameter Z. For example, if Z successively stands for utility, output, cost, revenue, and profit, we get isoutility function, isocost function, isorevenue function, and isoprofit function respectively.

The *IZF* : $Z=Z(L,K)$ is defined as the locus of various combinations of L and K, which shows the "isoness" of the parametric value of Z. The *IZF* for $Z=Z_0$ denoted by $Z_0 =Z(L,K)$ can be rewritten as $K=K(L, Z_0)$. For different parametric values of Z, say, $Z_1, Z_2, Z_3 \ldots \ldots Z_n$, the "iso-z-map" *(IZM)* can be shown in terms of the Figure 1.

Figure 1: Linear Iso-Z-Map



Source : Author's Construction

Further, the *IZF* assumes different shapes depending upon the sign of dK/dL and d^2K/dL^2, as follows:

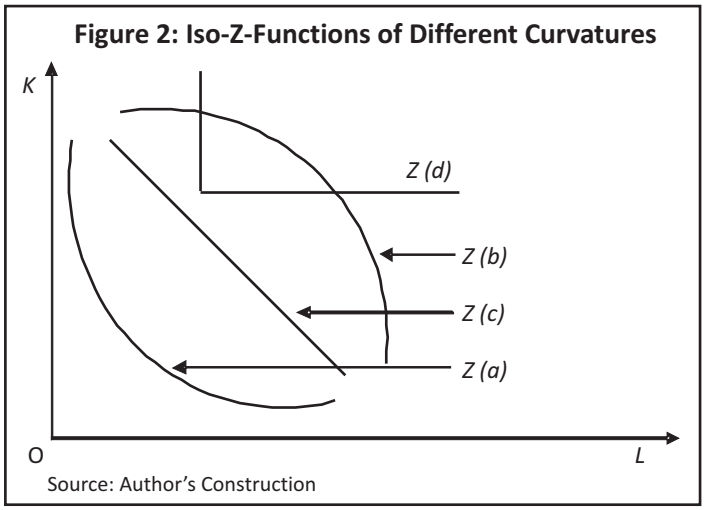
(a) $dK/dL<0$ and $d^2K/dL^2>0 \Rightarrow$ convexity of *IZF*.

(b) $dK/dL<0$ and $d^2K/dL^2<0 \Rightarrow$ concavity of *IZF*.

(c) $dK/dL<0$ and $d^2K/dL^2=0 \Rightarrow$ *negatively sloping linearity of IZF.*

(d) $dK/dL<0$ *and* $d^2K/dL^2=\infty \Rightarrow$ *L-shaped IZF.*

These four IZF_S are shown in the Figure 2.



Origin and Meaning of *ES*

The word "elasticity" is alien to economics. It has been imported form physics. Conflicting views about the introduction of "elasticity" into economics persist. The generally acceptable impression is that elasticity was introduced into economics in terms of Marshall's elasticity of demand. According to Stigler (1955), "Marshall was the first man to write about elasticity of demand = - (*dq/dp. p/q*)". Allen (1938/1979, p. 251) pointed out that "No established notation for elasticity is in current use".

If Marshall's "price elasticity of demand" is denoted by E_{DP} (sign ignored) for the demand function $P=P(D)$ such that $P'(D)<0$, then we get:

$E_{DP} = (dD/D) / (dP/P)$
$= (dD/dP) / (D/P)$
= (marginal demand) / (average demand)
$= (P/D) / (dP/dD)$
= (average demand price) / (marginal demand price)

In the early 1930s, Hicks (1932) and Robinson (1933), all about simultaneously and independently of one another and of their forerunners, struck out what came to be known as *ES*. Though they worked independently, they arrived at an identical result in the sense that by a curious coincidence, Hick's *ES* (*HES*) was exactly the same as Robinson's *ES* (*RES*).

If the isoquant (*IQ*) for Z_0 level of output is denoted by $Z_0=Z(L, K)$ or $K=K(L, Z_0)$, where L and K are two inputs and Z as a parameter stands for the level of output produced by the firm, then *HES* and *RES* can be denoted by the common notation σ, which can be defined as follows:

σ = (rate of change in input-ratio) / (rate of change in marginal physical product-ratio of inputs).

ES (σ) is of two types: **(i)** σ_{LK} and **(ii)** σ_{KL}, *where* $\sigma_{LK} \equiv ES$ of L for K and $\sigma_{KL} \equiv ES$ of K for L. If $k = (K/L)$, $l = (L/K) = (1/k)$, $m = (MP_L/MP_K) = (Z_L/Z_K)$, $n = (MP_K/MP_L) = (Z_K/Z_L) = (1/m)$, $MP_L \equiv$ marginal physical product of L and $MP_K =$ marginal physical product of K, then

(i) σ_{LK} *(along a given IQ)*
$= (dk/k)/(dm/m)$
$= (dk/dm)/(k/m)$
= (marginal *k*) / (average *k*)
$= (m/k)/(dm/dk)$
= (average *m*) / (marginal *m*)

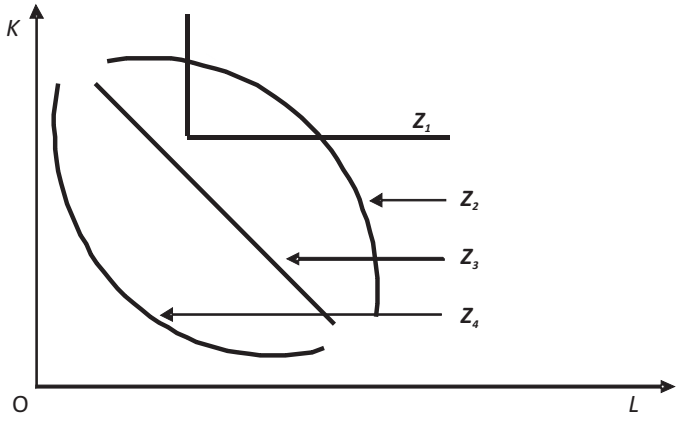
(ii) σ_{KL} (along a given *IQ*)

$= (dl/l)/(dn/n)$
$= (dl/dn)/(l/n)$
= (marginal l) / (average l)
$= (n/l)/(dn/dl)$
= (average n) / (marginal n)

Origin and Meaning of *SC*

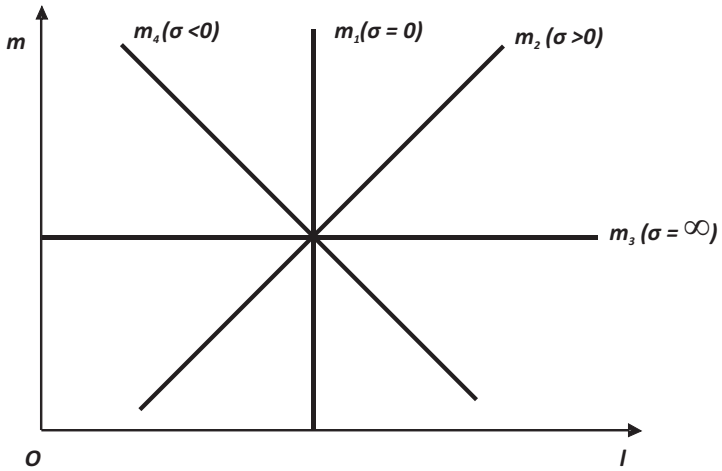
The concept of *SC* was introduced by Lerner (1933). By analogy of Marshall's demand curve, denoted by $P = P(D)$ such that $P'(D) < 0$ or its inverse $D = D(P)$ such that $D'(P) < 0$, Lerner devised the *SC* from a given *IQ*. As Marshall's demand curve *(MDC)* shows the functional relationship between the price (*P*) and demand (*D*), similarly the *SC* shows that between m ($= MP_L/MP_K$) or n ($= MP_K/MP_L$) and k ($= K/L$) or l ($= L/K$) given the *IQ*. Further, as *MDC* can symbolically be represented by $P = P(D)$ or its inverse $D = D(P)$, similarly, *SC* can be given by $m = m(l)$ or its inverse $l = l(m)$ given the *IQ*. The *SC* is the mapping of *IQ* from the input-space *(LK*-space*)* into the relative input – relative marginal physical product-space *(lm*-space*)*. According to Lerner (1933), *m* is the substitutability of *L* for *K*, while *n* is the substitutability of *K* for *L* along the same *IQ*. Following Lerner (1933), the relationship among the *IQ*, *SC*, and *ES* can be shown in terms of the Figures 3 and 4.

Figure 3: Isoquants of Varied Degrees of Curvature



Source: Author's Construction

Figure 4: Lerner's *SCs* of Varied *ESs* (σ)



Source: Author's Construction

Table 1: Similarity between Marshall's Demand Curve and Lerner's SC	
Marshallian Demand Curve (MDC)	**Lerner's SC**
$P \equiv$ (demand) price	$m = MP_L/MP_K \equiv$ substitutability of L for K.
$D \equiv$ quantity demanded	$l = L/K \equiv$ substitution or substitutability.
$P = P(D) \Rightarrow$ Marshall's demand function	$m = m(l) \Rightarrow$ Lerner's substitution function.
dP / dD = slope of *MDC*	dm / dl = slope of *SC*.
E_{DP} (sign ignored) = $(dD / dP) / (D / P)$ = marginal demand / average demand = $(P/D) /(dP / dD)$ = average (demand) price / marginal (demand) price	E_{lm} (sign ignored) = $(dl / dm) / (l / m)$ = marginal substitution / average substitution = $(m /l) / (dm/dl)$ = (average substitutability of L for K) / (marginal substitutability of L for K) = $HES = RES = \sigma$.
$E_{ij} \equiv$ elasticity of i with respect to j for the function $i= i(j)$	
Source: Author's Construction	

The *SC* denoted by m_i in Figure 4 is the mapping of *IQ* denoted by Z_i in the Figure 3 from *LK*-space into *lm*-space. Lerner (1933) pointed out that *ES* is inversely related to the curvature of the *IQ* $(= d^2K / dL^2)$, which is reflected in the shape and curvature of *SC* [i.e. $m'(l)$ and $m''(l)$]. He remarked that *ES* could be read off in the same way that the elasticity of demand is read off a demand curve and the elasticity of such a curve (*SC*) will give us the *ES*. The analogy between *SC* and *MDC* can be encapsulated in the Table 1. Needles to say, various types of *SC* or "similar curves" were developed later on by the economists like Kahn (1933), Sweezy (1933), Kennedy (1962), Matyas (1980), et al. But no vital difference was found among Lerner's (1933) *SC*, Kennedy's (1962) *SC* , and Matyas's (1980) "capital intensity function".

Origin and Meaning of *MRS*

Conventionally, it is received that the term *MRS* was used chronologically by Hicks and Allen (1934), Allen (1938/1979) and Hicks (1939). Such established notion is incorrect, because the concept of *MRS* was congealed or concealed in both *HES* (1932) and *RES* (1933), where *MRS* was treated as the "marginal physical product-ratio of inputs", that is, $MP_L/MP_K (= m)$ or $MP_K /MP_L (= n)$.

In modern terminology, *MRS* is the absolute slope of the *IZF* such as *IQ*, indifference curve or the similar curves. Like *ES*, *MRS* is also of two types: **(i)** MRS_{LK} and **(ii)** MRS_{KL}. While MRS_{LK} is read as *MRS* of L for K, MRS_{KL} is read as *MRS* of K for L.

So, $MRS_{LK} = |dK /dL| = MP_L/MP_K = Z_L / Z_K = m$ and $MRS_{KL} = |dL / dK| = MP_K /MP_L = Z_K / Z_L = n$, where the *IQ* is given by $Z_0 = Z (L, K)$ or $K=K (L, Z_0)$ and in the diagram of *IQ* map, L and K are measured along the horizontal and vertical axes respectively.

Identification and Correction of Terminological Inconsistencies of *ES*, *SC*, and *MRS*

According to temporal priority, *ES*, *SC*, and *MRS* can be considered successively as follows :

❖ Inconsistencies of *ES*

The following three inconsistencies are involved in the *ES* :

(1) *ES* was introduced by both Hicks (1932) and Robinson (1933) without the presumption of substitution function (*SF*) or substitution curve (*SC*). Because the elasticity of i with respect to j denoted by E_{ij} is pointless, if the i function given by $i= i(j)$ is not presumed. Marshall introduced the price elasticity of demand with the prior assumption of the demand function.

(2) The applicability of *ES* is confined to only the *IQ*, indifference curve and the similar curves. Even today's economists do not think that the *ES* may be applicable to the isocost, budget line, and the similar curves. Actually, *ES* can be applied to any kind of *IZF* irrespective of whether it is optimizing (or objective) *IZF* or constrained *IZF*.

(3) The notations $k\ (=K/L)$, $l\ (=L/K)$, $m\ (=MP_L/MP_K)$ and $n\ (=MP_K MP_L)$ have not been termed or defined fully by both Hicks (1932) and Robinson (1933) excepting that k or l stands for input- ratio and m or n stands for marginal physical product-ratio of inputs. Later on, consciously or unconsciously, k or l was termed as input-intensity, while m or n was termed as MRS_{LK} or MRS_{KL}, which also suffer from terminological inconsistencies.

❖ **Lerner's Unintended Correction of Inconsistencies of *ES* :** The foregoing three inconsistencies of *ES* were naively and partially corrected by Lerner (1933) with the introduction of his *SC*. Such introduction can be described as unintended or involuntary. Because his *SC* had been developed not to correct the foregoing inconsistencies of *ES*, but to establish the geometric relationship among *IQ*, *SC,* and *ES*, since he was not aware of such inconsistencies. By his unintended invention of *SC*, Lerner (1933) is assumed to correct the inconsistencies of *ES* naively and partially as follows:

(1) Introducing the *SC* denoted by $m = m\ (l)$ or $l = l\ (m)$, he ruled out the first inconsistency of *ES* as pointed out earlier.

(2) In the *SC* denoted by $m = m(l)$ or $l = l(m)$, m and n were respectively termed as substitutability of L for K and substitutability of K for L, while k and l were termed as simply substitution or substitutability. So Lerner's m or n is synonymous with the MRS_{LK} or MRS_{KL}. Through the creation of new terminology, he consciously or unconsciously corrected the third inconsistency of *ES* as pointed out earlier. By analogy of the Marshall's measurement of elasticity of demand with respect to price denoted by E_{DP} in terms of the demand curve, Lerner measured the *ES* by the elasticity of l with respect to m, denoted by E_{lm} in terms of *SC*.

Hence, E_{lm} (sign ignored) $= HES = RES = \sigma$
$= (dl/l)/(dm/m)$
$= (dl/dm)/(l/m)$
$=$ (marginal l) / (average l)
$=$ (marginal substitution) / (average substitution)
$= (m/l)/(dm/dl)$
$=$ (average substitutability of L for K) / (marginal substitutability of L for K)

Thus we find that *HES* or *RES* (σ) measured along the given *IQ* is identical with the E_{lm} along the Lerner's (1933) *SC*. By analogy of Marshall's method of price elasticity of demand, Lerner corrected the first inconsistency of *ES* as pointed out earlier.

❖ **Identification of Lerner's (1933) own Inconsistencies**

(1) Lerner's *SC* had not been applied to the diverse aspects of firm's behaviour as Konar's (2009, 2012) "isoquant substitution curve" *(IQSC)* and "isocost substitution curve" *(ICSC)* can be applied.

(2) In Lerner's *SC* denoted by $m = m(l)$ or $l = l(m)$, l is termed as simply substitution or substitutability. But since substitution is likened to a two-way-traffic, so why should l be interpreted simply as substitution or substitutability, not as substitution or substitutability of K or L for L or K ? Following Lerner's line of thought, l should be treated as substitution or substitutability of K for L. Thus, we get the result that while m stands for substitutability of L for K, then l stands for substitutability of K for L. These results are also inconsistent. Hence, m should be read as marginal substitution of L for K, n as marginal substitution of K for L, k as average substitution of L for K and l as average substitution of K for L.

(3) From Lerner's SC denoted by $m = m(l)$ or $l = l(m)$, what we get by E_{lm} is the "elasticity of substitution *(l)* with respect to substitution *(m)*". Is this not inconsistent? If Marshall were alive today, would he say "elasticity of demand with respect to demand" by E_{DP} ? At best, he would say "elasticity of demand with respect to demand price" for the demand function given by $P = P(D)$ or its inverse $D=D(P)$ such that $P' < 0$ or $D' < 0$. In fact, for the Lerner's *SC* given by $m = m(l)$ or its inverse $l = l(m)$, E_{lm} (sign ignored) should be read as "elasticity of average substitution of K for L with respect to marginal substitution of L for K".

❖ **Inconsistencies of *MRS* :** When *MRS* was introduced, the foregoing inconsistencies of *ES* and Lerner's *SC* were naively and partially mended unconsciously or involuntarily for the simple reason that *MRS* was brought in not to rule out these intrinsically subtle and concealed inconsistencies, as these inconsistencies had been disclosed by none of our forerunners. With the introduction of *MRS*, which is of two types: $|dK/dL| = m = MP_L/MP_K = MRS_{LK}$ *and* $|dL/dK| = n = MP_K/MP_L = MRS_{KL}$, one constituent element $m\ (=MP_L/MP_K)$ or $n\ (= MP_K/MP_L)$ of *ES* (another constituent element of *ES* is *l* or *k*) came into explicit form from implicit form of *MRS*. So *m* or *n* in *ES* remained no longer undefined since the introduction of *MRS*. Of the two constituent elements in *ES* such as *m* (or *n*) and *l* (or *k*), the residual elements *l* (or *k*) still remained undefined or unexplained, though the entanglement of *m* or *n* was removed erroneously with the introduction of *MRS*, since *MRS* itself was inconsistent. Hence, *m, n, k,* and *l* should be redefined or re-explained.

Now let us see what should have happened with the introduction of *MRS*. For the *IQ* denoted by $Z_0 = Z(L,K)$ or $K=K(L,Z_0)$, if $|dK/dL| = MP_L/MP_K$ and $|dL/dK| = MP_K/MP_L$ are termed as MRS_{LK} and MRS_{KL} respectively, so why should $k\ (=K/L)$ and $l\ (= L/K)$ not be termed as ARS_{LK} and ARS_{KL} respectively?, where *ARS* stands for "average rate of substitution". Because, in the definition of "absolute elasticity of *K* with respect to *L*" along the *IQ* denoted by $|E_{KL}| = |(dK/K)/(dL/L)| = |(dK/dL)|/(K/L) = MRS_{LK}/(??)$, what term will be appropriate for *K/L* in the denominator marked by (??), while the term MRS_{LK} is put into the numerator. Following Hicks's way of thought, it can be argued that *K/L* must be termed as ARS_{LK} and *L/K* as ARS_{KL}. If this is so (though erroneous), then we have:

$HES = RES = \sigma = E_{lm} = E_{(ARS)(MRS)}$

= (marginal substitution) / (average substitution)
= *(dl/dm) / (l/m)*
= *{d (ARS)/ d (MRS)}/ (ARS/MRS)*
= *(*marginal *ARS) / (*average *ARS)*
= *(m/l) / (dm/dl)*
= *(MRS/ARS) / {d(MRS) / d(ARS)}*
= *(*average *MRS) / (*marginal *MRS)*

Further, since marginal function and average function are the two derivatives or derived functions of total or original function, so if *dK/dL* (sign ignored) = MRS_{LK} and *K/L* = ARS_{LK}, then what will be TRS_{LK} (total rate of substitution)? Following Hicks's line of thought (though erroneous), it can be pointed out that $K=K\ (L, Z_0)$ should be termed as TRS_{LK} and $L=L\ (K, Z_0)$ as TRS_{KL}. In addition to the foregoing inconsistencies of *MRS*, the residual inconsistencies of *MRS* is that *MRS (ARS* and *TRS* too) is confined to only *IQ*, indifference curve and other similar curves and it was or is not applied to isocost, budget line, and other similar IZF_S.

Mending the Terminological Inconsistencies of *ES, SC,* and *MRS*

The foregoing inconsistencies can be mended or ended in the following ways:

❖ **Isoquant (IQ) :** *Let* $Z_0 = Z(L,K)$ *or* $K = K(L,K_0)$ be the production function *(PF)* representing the *IQ* for $Z = Z_0$ level of output. Since the *PF* shows the "technical relationship" between the inputs and output, so this "technical nature" of the *PF* should be reflected in its derivative functions, as follows:

(a) $K = K(L,Z_0)$ should be termed as total *IQ (TIQ)* or "total technical substitution" *of L* for *K* (TTS_{LK}).

(b) $|dK/dL| = MP_L/MP_K = m$ (say) = absolute slope of *TIQ*, which should be termed as absolute marginal *IQ* (*AMIQ*) or "marginal technical substitution" of *L* for *K* (MTS_{LK}).

(c) $(K/L) = k$ = slope of the ray drawn from the origin to a point on *TIQ*, which should be termed as average *IQ* (*AIQ*) or "average technical substitution" of *L* for *K* (ATS_{LK}).

(d) $|E_{LK}| = |(dK/K)/(dL/L)| = (|dK/dL|)/(K/L) = MIQ/AIQ = MTS_{LK}/ATS_{LK} = m/k$ = absolute elasticity of *K* with respect to *L* along the *TIQ*.

(e) E_{km} (sign ignored) = $(dk/k)/(dm/m) = (dk/dm)/(k/m)$ = marginal AIQ or ATS_{LK} / average AIQ or ATS_{LK} = elasticity of k with respect to m along the Lerner's SC or Konar's (2009) *isoquant substitution curve* ($IQSC$) ≡ $\sigma_{LK} = HES = RES$ along TIQ. Noteworthy that E_{km} should be substituted for σ_{LK}, which suffers from multitude of inconsistencies.

(f) $L = L(K, Z_0)$ should be read as inverse TIQ or $TTS_{LK} = TTS_{KL}$.

(g) $|dL/dK| = MP_K/MPL = n$ (say) should be read as inverse $AMIQ$ or $MTS_{LK} = MTS_{KL}$.

(h) $(L/K) = l$ (say) should be read as inverse AIQ or $ATS_{LK} = ATS_{KL}$

(i) $|E_{LK}|$ (along the TIQ) = $|(dL/L)/(dK/K)| = (|dL/dK|)/(L/K) = MTS_{KL}/ATS_{KL}$.

(j) E_{ln} (sign ignored) [along Lerner's (1933) SC or Konar's (2009) *isoquant substitution curve*] = $(dl/dn)/(l/n)$ = marginal ATS_{KL} / average $ATS_{KL} = \sigma_{KL}$ *(=HES = RES along TIQ).*

❖ **Isocost (IC) :** *Let* $C_0 = (P_L.L + P_K.K) = C(L, K)$ *or* $K = K(L, C_0)$ be the cost function representing the IC for $C = C_0$ level of cost of production incurred by the firm. Since the IC shows the "economic relationship" between the inputs and the cost of production, so its "economic nature" should be reflected in its derivative functions, as follows:

(a) $K = K(L, C_0)$ should be renamed as total *IC (TIC) or "total economic substitution" of L for K* (TES_{LK}).

(b) $|dK/dL| = MC_L/MC_K = P_L/P_K = P$ (say) should be termed as absolute marginal IC ($AMIC$) = "marginal economic substitution" of L for K (MES_{LK}) = absolute slope of TIC or TES_{LK}, where MC_L and MC_K respectively stand for marginal costs of L and K.

(c) $K/L = k$ (say) should be termed as average isocost (AIC) or "average economic substitution" of L for K (AES_{LK}) = slope of the ray drawn from the origin to a point on TIC.

(d) $|E_{KL}|$ (along the TIC) = $|(dK/K)/(dL/L)| = (|dK/dL|)/(K/L) = AMIC/AIC = MES_{LK}/AES_{LK}$ = absolute elasticity of K with respect to L along the TIC.

(e) E_{kp} (sign ignored) = $(dk/k)/(dP/P) = (dk/dP)/(k/P)$ = marginal AIC or AES_{LK}/average AIC *or* AES_{LK} = elasticity of k with respect to P ($= P_L/P_K$) along the Konar's (2009) *isocost substitution curve* ($ICSC$). Noteworthy that $HES = RES$ (σ) was applicable to only IQ, but it can also be applied to IC irrespective of its curvature.

(f) $L = L(K, C_0)$ should be termed as inverse TIC or $TES_{LK} = TES_{KL}$.

(g) $|(dL/dK)| = MC_K/MC_L = P_K/P_L$ should be read as inverse $AMIC$ or $MES_{LK} = MES_{KL}$.

(h) $L/K = l$ (say) should be read as inverse AIC or $AES_{LK} = AES_{KL}$.

(i) $|E_{LK}|$ (along the TIC) = $(|dL/dK|)/(L/K) = MES_{KL}/AES_{KL}$.

Concluding Comments

Konar's (2009, 2012) $IQSC$ and $ICSC$ are the two tools which can be conceived as the mender of last resort of the terminological inconsistencies of ES, SC, and MRS. Further, these two tools can also be applied to the conventional and controversial areas of the theory of firm's behavior, as has been executed by Konar (2012). The potential researchers can construct different "iso$Z$ substitution curves" (e.g. isoutility substitution curve, isobudget substitution curve, isorevenue substitution curve, isoexpenditure substitution curve, etc.) for realizing their different roles and results, where Z may stand for isoutility curve, isobudget curve, isorevenue curve, isoexpenditure curve, etc.

> "Every invention of an idea can be seen *post hoc* to solve a problem or answer a question. Said another way, there may not be an answer for every question, but there is a question for every answer and similarly, there may not be a solution for every problem, but there is a problem for every solution" (Boland, 1994).

There is hardly any research, which is free from criticism. The present paper also is not devoid of potential criticisms. But the "mode of criticism" should be guided by "Popper's principle of sympathetic problem orientation". Such

principle implies that the critic must indicate the researcher's problem and solution, but only after making every effort to present the researcher's views in the "most sympathetic light". That is, the critic must make all unchallengeable improvements that can be made before launching the criticism. One should not wish to distract the debate into irrelevant side issues. In effect, the criticism must be conducted in terms that the researcher can accept (Boland, 1994).

References

Allen, R. G. D. (1938/1979). *"Mathematical Analysis for Economists."* London, Macmillan Press.

Allen, R. G. D., & Mishan, E. J. (1965). "Is the Substitution Term Ambiguous?" *Economica , 32* (125), pp. 215-222.

Boland, L. (1994). "Scientific Thinking without Scientific Method: Two Views of Popper." In Backhouse, R. (Ed.), *"New Directions in Economic Methodology."* London, Routledge.

Chipman, J. S. (1960). "The Foundations of Utility." *Econometrica, 28* (2), pp. 193-224.

Folk, H. W. & Wolfe, J. N. (1964). "The Ambiguity of the Substitution Term." *Economica, 31* (3), pp. 288-293.

Heath, T. L. (Ed.)[n.d.]. "On the Sphere and Cylinder." In *"The Works of Archimedes."* Vol. 1, New York, Dover Publications, p. 4.

Hicks, J.R. (1932). *"Theory of Wages."* New York, Macmillan.

Hicks, J.R. (1939). *"Value and Capital."* London, Oxford University Press.

Hicks, J. R., & Allen, R. G. D. (1934). "A Reconsideration of the Theory of Value." *Economica, 1* (2), pp. 196-219.

Kahn, R. F. (1933). "The Elasticity of Substitution and the Relative Share of a Factor." *Review of Economic Studies, 1* (1), pp. 72-78.

Kennedy, C. (1962). "The Character of Improvements and of Technical Progress." *Economic Journal, 72* (328), pp. 899-911.

Konar, A. K. (2009). "Surrogate Substitution Curve of Lerner (1933)." *Asian Economic Review, 51* (1), pp. 133-146.

Konar, A. K. (2012). "New Applications of Konar's (2009) Isoquant Substitution Curve and Isocost Substitution Curve." *Asian Economic Review, 54* (2), pp. 367-382.

Lerner, A.P. (1933). "Notes on Elasticity of Substitution: The Diagrammatical Representation." *Review of Economic Studies*, *1* (1), pp. 68-71.

Matyas, A. (1980). *"History of Modern Non- Marxian Economics."* London, Macmillan.